Central University of Haryana
ODD Semester Term End Examination April 2022
B.Tech. Program
Branch: B. Tech. (Computer Science and Engineering)

Course Code: BT CS 403 **Max Time: 3 Hours**
Course Title: Discrete Structures **Max Marks: 70**

Instructions:

Question Number **one (PART-I)** is compulsory and carries total 14 marks, attempt all seven sub-parts. (Each sub part carries two Marks).

Question Numbers 2(two) to 5(five) carry fourteen marks each.

PART -I

Q. No.1 (2 X 7 = 14)

(a) Find the conjunction of the propositions p and q where p is the proposition "Rebecca's PC has more than 16 GB free hard disk space" and q is the proposition "The processor in Rebecca's PC runs faster than 1 GHz."

(b) What type of sentence is 5+x =9? For what value of x it will become a true statement.

(c) Show that the inclusion relation ⊆ is a partial ordering on the power set of a set S.

(d) Let P (x) denote the statement "x > 3." What are the truth values of P (4) and P (2)?

(e) How many ways are there to arrange the eight letters in the word CALCUTTA?

(f) Define the term Semigroup

(g) Define tree and its properties.

PART –II

Q. No.2

(i) Define the reflexive, symmetric, and transitive closure of a relation. Let R = {(1,2), (2,3), (3,1)} and A = {1, 2, 3}, find the reflexive, symmetric, and transitive closure of R. (7)

(ii) Design the compatible total ordering for the poset ({1, 2, 4, 5, 12, 20}, /). (7)

OR

Q. No. 2

(i) Solve the recurrence relation $a_{n+2} - a_{n+1} - 2a_n = n^2$. (7)

(ii) Solve the recurrence relation $a_n - 4a_{n-1} + 4a_{n-2} = 1, \forall n \geq 2 \; with \; a_0 = 0, a_1 = 1$. (7)

Q. No.3

(i) Construct the truth table of the compound proposition $(p \vee \sim q) \rightarrow (p \wedge q)$. (4)

(ii) Define the contrapositive of the conditional statements. Prove the theorem, "If n is an integer, then n is odd if and only if n^2 is odd. (6)

(iii) Show that if n is a positive integer, then $1 + 3 + \cdots + (2n - 1) = n^2$ (4)

OR

Q. No 3

(i) A committee of 5 is to be formed out of 6 males and 4 females. In how many ways this can be done when (i) at least 2 females are included (ii) at most 2 females are included. (4)

(ii) What is the minimum number of students required in a discrete mathematics class to be sure that at least six will receive the same grade, if there are five possible grades, A, B, C, D, and F? (5)

(iii) How many poker hands of five cards can be dealt from a standard deck of 52 cards? Also, how many ways are there to select 47 cards from a standard deck of 52 cards? (5)

Q. No.4

(i) Show that set of all non-zero real numbers is a group with respect to multiplication. (6)

(ii) Ex. Let (Z, *) be an algebraic structure, where Z is the set of integers and the operation * is defined by $n * m = maximum\ of\ (n, m)$.

I) Show that $(Z,*)$ is a semi group. (4)

II) Is $(Z,*)$ a monoid?. Justify your answer. (4)

OR

Q. No .4

(i) Show that the set of all strings 'S' is a monoid under the operation 'concatenation of strings'. Is S a group w.r.t the above operation? Justify your answer. (7)

(ii) Let R be a group of all real numbers under addition and R^+ be a group of all positive real numbers under multiplication. Show that the mapping $f : R^+ \longrightarrow R$ defined by $f(x) = \log_{10} x$ for all $x \in R$ is an isomorphism. (7)

Q. No.5

(i) Prove that the maximum number of vertices on level n of a binary tree is 2^n where $n \geq 0$. (4)

(ii) Define the terms (10)

1. Spanning tree
2. Connected and strongly connected graph
3. In-degree and out-degree
4. Subgraph
5. Path

OR

Q. No.5

(i) Prove that a graph is bipartite if and only if all its circuits are of even length. (7)

(ii) A tree has two vertices of degree 2, one vertex of degree 3 and three vertices of degree 4. How many vertices of degree 1 does it have? (7)