CENTRAL UNIVERSITY OF HARYANA
Third Semester Examinations Jan-2023

Programme	: Integrated B.Sc.-M.Sc. (Mathematics)	Session	: 2022-2023
Semester	: III	Max. Time	: 3 Hours
Course Title	: Group Theory	Maximum Marks	: 70
Course Code	: SBSMAT 03 03 02 C 5106		

Instructions:

1. Question no. 1 has has seven sub parts and students need to answer any five. Each sub part carries two marks.
2. Question no. 2 to 6 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries six marks.

1. (a) Define group. Give an example of non-abelain group with eleboration.
   (b) Show that in a group there is only one identity.
   (c) Define order of an element. Consider $U(15)$ under multiplication modulo 15, find the order of 7, 11 and 13.
   (d) Show that any two disjoint permutations commute.
   (e) Show that center of a group G is a subgroup of G.
   (f) How $U(8)$ is not a cyclic group.
   (g) Define Ring. Give an example of commutative ring with unity and mention the unity.

2. (a) Show that a finite semi-group in which cancellation laws hold is a group.
   (b) Show that symmetries of a square forms a group. Is it commutative.?
   (c) Define quaternion. Show that quaternions form a group.

3. (a) A non-empty subset H of a group G is a subgroup of G if and only if $ab^{-1} \in H$, whenever $a, b \in H$.
   (b) Define Euler phi function. State and prove the Euler's theorem.
   (c) State and prove Lagrange's theorem. Is the converse of Lagrange's theorem true justify your statement.

4. (a) A subgroup H of a group G is normal subgroup of G if and only if the product of two right cosets of H in G is again a right coset of H in G.
   (b) Let G be a finite group and suppose p is a prime such that $\frac{p}{o(G)}$, then there exits $x \in G$ such that $o(x) = p$.
   (c) Show that $N(x^{-1}ax) = x^{-1}N(a)x$, for all $a, x \in G$.

5. (a) Define permutation. Show that an odd permutation is of even order.
   (b) Show that every permutation can be written as a product of disjoint cycles.
   (c) Show that every group is isomorphic to a permutation group.

6. (a) Define isomorphism. If $f : G \longrightarrow G'$ is a homomorphism then show that $f(e) = e'$ and $f(x^{-1}) = (f(x))^{-1}$.
   (b) Show that every homomorphic image of a group G is isomorphic to a quotient group of G.
   (c) State and prove the third theorem of isomorphism.