

# CENTRAL UNIVERSITY OF HARYANA

Term End Examinations January 2023

Programme: Integrated B.Sc.-M. Sc. (Mathematics)

Session: 2022-23

Semester: Third

Max. Time: 3 Hours

Course Title: Probability & Statistics

Max. Marks: 70

Course Code: SBSMAT 03 03 03 C 5106

---

## Instructions:

1. Question no. 1 has seven parts and students are required to answer any five. Each part carries two marks.
2. Questions no. 2 to 6 have three questions and students are required to answer any two parts of each question. Each part carries six marks.

Q 1. (5X2=10)

- a) Let A and B be two events such that  $P(A) = 0.3$  and  $P(A \cup B) = 0.8$ . If A and B are independent events then  $P(B) = \dots\dots$
- b) Write the probability axioms.
- c) The variance of the binomial distribution  $\binom{10}{x} \left(\frac{2}{3}\right)^x \left(\frac{3}{5}\right)^{10-x}$ ;  $x = 0, 1, 2, \dots, 10$  is  $\dots\dots$
- d) Define moment-generating function.
- e) Define weak law of large numbers (WLLN).
- f) The joint pdf of random variables in bivariate normal distribution is  $\dots\dots$
- g) A coin is biased so that the probability of head is 0.60. The entropy is  $\dots\dots$

Q 2. (2X6=12)

- a) State and prove Baye's theorem.
- b) The diameter of an electric cable, say X, is assumed to be a continuous random variable with :  $f(x) = 6x(1-x)$ ,  $0 \leq x \leq 1$ .
  - (i) Check that f(x) is p.d.f. and
  - (ii) Determine a number b such that  $P(X < b) = P(X > b)$ .
- c) What do you mean by moment generating function? A coin is tossed until a head appears. What is the expectation of the number of tosses required?

Q3. (2X6=12)

- a) A manufacturer, who produces medicine bottles, finds that 0.1% of the bottles are defective. The bottles are packed in boxes containing 500 bottles. A drug manufacturer buys 100 boxes from the producer of bottles. Using Poisson distribution, find how many boxes will contain: (i) no defective, and (ii) at least two defectives. [ Given  $e^{-0.5} = 0.6065$  ]
- b) Show that the mean deviation from the mean of the normal distribution is about 4/5 of its standard deviation.

- c) Derive expressions for the mean and variance of Chi-Square distribution.

(2X6=12)

Q 4.

- a) A two-dimensional random variable (X, Y) have a bivariate distribution given by:

$P(X = x, Y = y) = \frac{x^2 + y}{32}$ , for  $x = 0, 1, 2, 3$  and  $y = 0, 1$ . Find the marginal distributions of X and Y.

- b) The joint probability density function of a two-dimensional random variable (X, Y) is

given by:  $f(x, y) = \begin{cases} 2; & 0 < x < 1, 0 < y < x \\ 0 & \text{elsewhere} \end{cases}$

- (i) Find the conditional density function of Y given X = x and conditional density function of X given Y = y.

- (ii) Check for independence of X and Y.

- c) If X and Y are continuous random variables, then show that  $E(X+Y) = E(X) + E(Y)$ .

(2X5=10)

Q 5.

- a) Calculate coefficient of rank correlation from the following data:

X	15	10	20	28	12	10	16	18
Y	16	14	12	12	11	15	18	12

- b) Define covariance of random variables. State and prove at least three properties of covariance.

- c) State and prove Chebyshev's theorem.

(2X6=12)

Q 6.

- a) What do you mean by information? Discuss in detail about component of the information.

- b) Show that  $H(X, Y) = H(X) + H(Y/X)$ .

- c) Discuss in detail about Polya's urn model.